Chapter 1

A two-dimensional rough surface: Experiments on a pile of rice.

C. M. Aegerter, R. Günther, and R. J. Wijngaarden
Department of Physics and Astronomy; Vrije Universiteit; 1081HV Amsterdam; The Netherlands.

Dynamical roughening of interfaces has received much attention in recent years. However, experiments have been restricted to one dimensional (1d) systems. Moreover, theoretical studies of the two dimensional (2d) case have been highly inconclusive. Here we introduce an experimental 2d system, with which the theories can be tested. As is shown, the surface of a 2d pile of rice shows roughening behaviour in both space and time, with a roughness exponent $\alpha_{2d} = 0.39(3)$ and a growth exponent $\beta_{2d} = 0.27(3)$.

1.1 Introduction

Roughening phenomena of interfaces have been studied extensively in recent years due to their wide range of applicability. Rough interfaces appear in such diverse systems as flux propagation in superconductors [1], the burning of papers [2], diffusion waves [3], bacterial colonies [4], flow through porous media [5] and many more [6]. Even though all of these systems have very different microscopic physics governing the processes, they can be described by simple models from a very small number of universality classes [6]. The most famous such model is described by a non-linear diffusion equation known as the Kardar-Parisi-Zhang (KPZ) equation [7]:

$$\partial_t h(x,t) = \nu \Delta h(x,t) + \lambda (\nabla h(x,t))^2 + \eta(x,t), \qquad (1.1)$$

where ν is the diffusion coefficient, η is a noise term, λ quantifies the non-linearity and $h(x,t)$ is the position of the interface. In one dimension, the scaling behavior of an interface governed by the KPZ equation can be analytically solved. The roughness is parameterized by the width of the interface given by:

$$\sigma(t,L) = \left(\langle(h(x,t) - \langle h(t)\rangle_L)^2\rangle_L\right)^{1/2}. \tag{1.2}$$

Here, $\langle\cdot\rangle_L$ denotes the average over the interface in space. For a self-affine surface, the width growth as a power law in time $\sigma \sim t^\beta$, until saturation is reached when the correlation length becomes comparable to the system size [6]. This growth exponent, β, characterizes the dynamics of the process. After the saturation time, the width is constant in time at a value $\sigma_{sat}(L) \sim L^\alpha$, which grows as a power law with the system size [6]. This roughness exponent, α, characterizes the structure of the interface. For the KPZ equation, one obtains $\alpha = 1/2$ and $\beta = 1/3$ [7].

For a multi-dimensional KPZ system however, the theoretical situation is unclear. Analytical treatments of the KPZ equation only exist in approximations [8, 9, 10], and results from numerical simulations vary greatly [6]. Similarly, experiments have up to now been restricted to a single dimension. The experimental problem of a two dimensional rough surface asks for a surface reconstruction technique with enough spatial resolution to span some orders of magnitude, while at the same time having the temporal resolution to capture the dynamics of the process, which is not easily achieved. Secondly, a system has to be found that exhibits KPZ roughening in 2d and is accessible experimentally. Presently, there is some interest to combine the exact results on the KPZ equation with the concept of self-organized criticality (SOC) [11]. The surface of a sandpile, which is the archetypal system to study SOC, can be mapped onto a system which follows KPZ dynamics [12]. This is intriguing since it brings together two established fields of research, however has not yet been tested experimentally. We study here the front of a 2d rice-pile and its roughening behavior, showing that it does indeed obey KPZ dynamics. We choose rice, since it has been shown in 1d that a rice-pile does indeed show SOC behaviour [13]. With the surface of a rice-pile established as a roughening system, we can extend the study further to include the full 2d surface of the pile spanning an area of ~1x1m^2. Using this system, we can determine roughening and growth exponents in 2d and compare them with theoretical predictions [8, 9, 10].

In section 2, we will discuss the experimental setup including the surface reconstruction technique based on active-light stereoscopy, as well as the growing mechanism of the pile. The analysis techniques are developed in section 3, with special emphasis on the generalization from known 1d methods to the 2d problem, while in section 4 the results of the rice-pile experiment are presented. There, it is first shown that the front behaviour in 1d does in fact obey KPZ scaling before discussing the 2d results. Those results are used to put constraints on theoretical results for 2d KPZ behavior.

1.2 Experimental setup

The rice-pile is grown by dropping rice, uniformly distributed along a line using a custom-built dispenser. The dispenser consists of a distribution board and a sowing machine. In the sowing machine, an eccentric rotor keeps the rice in motion such that a steady flow of rice is achieved at the rate of ~5 g/s. This flow of rice is subsequently distributed along a line of 1 m in the distribution board using simple geometry (see Fig. 1.1). The principle is related to that of a pin-board producing a Gaussian distribution.

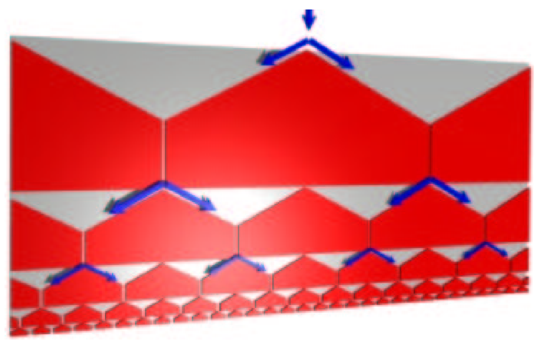

Figure 1.1: A schematic image of the distribution board. Rice is dropped from a single point on the top and subsequently divided into even compartments. At the end a line of rice uniformly distributed in 64 intervals is obtained, which is used to grow the rice-pile at a rate of ~5 g/s.

In order to study the surface properties of the rice-pile, a 3d reconstruction technique was developed, based on active-light stereoscopy [14]. A set of colored lines is projected onto the pile at approximately right angles using an overhead projector. In the stereoscopic view [15], the projector takes the place of the second camera passing its information to the camera via the colored lines. The camera itself is placed at an angle of 45 degrees to the surface of the pile and the projected lines. From this view-point the projected lines are deformed and can be used to determine the 3d structure of the surface in the same way as iso-height-lines do on a map. An example of such a reconstruction is shown in Fig. 1.2. Measurements on test objects show that a surface of 1x1 m^2 can be reconstructed with an accuracy of 1-2 mm, which is comparable to the size of the rice grains and thus suited for the present purpose. The use of differently colored lines allows for better filtering and thus for better identification of the lines in the computer.

1.3 Analysis methods

As noted in the introduction, rough surfaces are often analyzed using the width of the interface to characterize its structure and dynamics. However, in order to

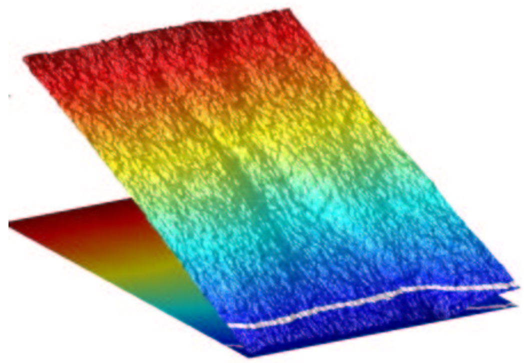

Figure 1.2: Reconstruction of the surface of a rice-pile. The white line indicates the position of the growing front.

obtain reliable results, many experiments have to be averaged over, using such a method [6]. A more promising way of analysis, which has been extensively used in the analysis of 1d experiments is via the two-point correlation function [6]

$$C(x,t) = \langle (h(\xi,\tau) - h(\xi+x,\tau+t))^2 \rangle_{\xi,\tau}^{1/2}. \tag{1.3}$$

In both space and time the scaling behaviour of the correlation function is the same as that of the width thus making it possible to determine the growth and roughness exponents from $C(x,t)$. In addition, the growth exponent can be determined from data obtained after the saturation time, since in the correlation function only time differences are important.

When generalizing the method to 2d, computational difficulties arise. Because of the number of points to compare, the number of operations to be carried out to determine the correlation function grows with the fourth power of the size of the surface. However, tests on small surfaces indicate that the radial average of $C(x,y,t)$ scales like the 2d local width, but due to the computational inefficiency we were using yet another method to determine the roughness and growth exponents.

The power spectrum, or structure function [16], can be determined easily for 1d and 2d systems from the square of the Fourier transform $\hat{h}(k_x,k_y)$ of the local height $h(x,y)$

$$S(k_x,k_y) = |\hat{h}(k_x,k_y)|^2. \tag{1.4}$$

Here, the computational load is just given by determining the Fourier transforms, which also in 2d only grows with the square of the size of the surface thus making it feasible to calculate the distribution function of the whole rice-pile surface. The square root of the integral of $S(k_x,k_y)$ over k-space, the distribution function $\sigma(k_x,k_y)$,

$$\sigma(k_x,k_y) = \left(\int \int S(k_x,k_y) dk_x dk_y \right)^{1/2} \tag{1.5}$$

is equal to the rms-width of the interface [16]. Thus the distribution function also has the same scaling behaviour as the width and can therefore be used to determine the roughness and growth exponents. Again, a radial average of $\sigma(k_x, k_y)$, $\sigma(k)$, scales like the 2d local width, thus making comparisons with previous simulations of ballistic deposition models possible. Moreover, real 2d measures like the distribution function can also give information about anisotropies of the scaling in the x- and y-directions.

The distribution function is also useful in investigations of the dynamics of the processes. In that case, the square of the Fourier transform $\hat{h}(\omega)$ of the time dependence $h(t)$ has to be determined. The Fourier transforms are determined using an FFT algorithm after padding the data with zeros to the next power of two.

1.4 Results and discussion

In order to determine that the rice-pile surface does in fact follow KPZ behaviour we first determined the roughness and growth exponents of the front of the pile, given by the line of equal height of the pile at 0.1 m. The distribution func-

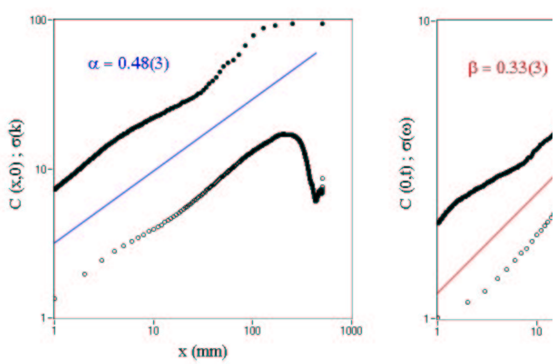

Figure 1.3: The behavior of the front of a propagating rice-pile. Both the correlation function (open symbols) and the distribution function (full symbols) show scaling in space and time over two decades. The resulting roughness and growth exponents are in excellent agreement with the KPZ universality class.

tions determined in both space, $\sigma(k)$, and time, $\sigma(\omega)$, as well as the correlation functions $C(x,t)$ can be seen in Fig. 1.3, where the values of $\alpha = 0.48(3)$ and $\beta = 0.33(3)$ can be inferred. These values are in excellent agreement with the expectations from the KPZ equation thus establishing that KPZ behaviour does appear in SOC systems.

The 2d distribution function, $\sigma(k_x, k_y)$, which characterizes the roughening of the whole surface is shown in Fig. 1.4 on a triple-logarithmic plot. In the insert, the angular dependence of a power-law fit to σ is shown. This indicates a dependence of the roughness exponent α on the direction, which shows the anisotropy of the system. Such an anisotropy most probably arises from the growth mechanism of the pile, which is seeded from a horizontal line, thus breaking the symmetry of the x- and y-directions. It should be noted here that the exponents corresponding to the x- and y-directions do not have to agree with those determined in a 1d analysis. This is because $\sigma(k_x, 0)$ already includes data from the y-direction due to the complex nature of the Fourier transform. Thus, $\sigma(k_x, 0)$ already presents an effective 2d measure.

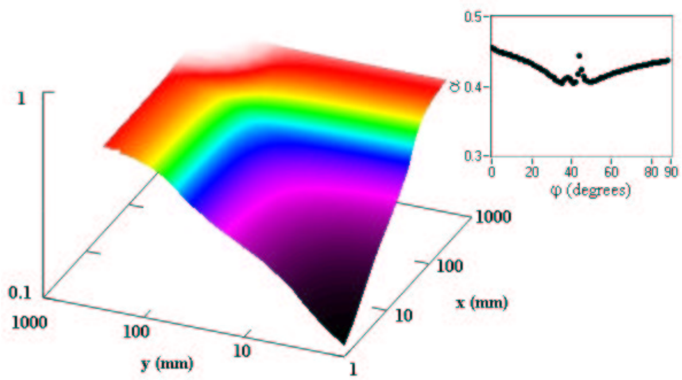

Figure 1.4: The 2d distribution function for the rice-pile surface on a triple-logarithmic plot. From a radial average, the roughness exponent can be determined. The insert shows an angular dependence of the roughness exponent, with an anisotropy in the x- and y-directions.

The radial average of the distribution function, $\sigma(k)$, is shown in Fig. 1.5a, where the value of the roughness exponent can also be determined. We obtain $\alpha_{2d} = 0.39(3)$, which is also in agreement with the average of the exponents determined as a function of angle. In addition, the temporal behavior, $\sigma(\omega)$ is shown in Fig. 1.5b, where we determine the 2d growth exponent to be $\beta_{2d} = 0.27(3)$. Both the roughness and growth exponents determined experimentally are in very good agreement with the conjecture derived from solid-on-solid models by Kim and Kosterlitz [17] for higher dimensional exponents given by $\alpha = 2/(d+3)$ and $\beta = 1/(d+2)$. Numerical results from integrating the 2d KPZ equation vary greatly, with values of α_{2d} ranging from 0.18 [10] to 0.39 [8] and β_{2d} ranging from 0.1 [10] to 0.25 [8]. Our experimental results are in good agreement with the numerical values of Amar and Family [8], as well as Bouchaud and Cates [9] corresponding to the high range of the values, while excluding most of the other numerical investigations into 2d KPZ behavior.

1.5 Conclusions

We have presented an experimental study on roughening in a 2d system. The surface of a rice-pile is measured with a reconstruction technique based on active-

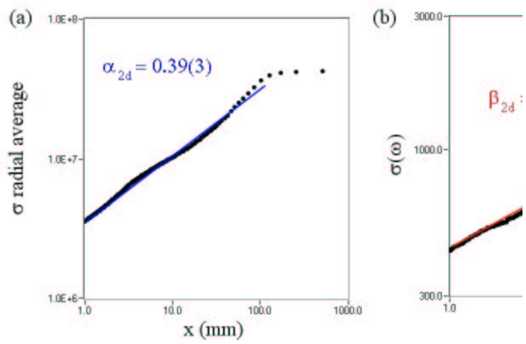

Figure 1.5: (a) The radial average of the 2d distribution function, allowing the determination of the roughness exponent in 2d to be $\alpha_{2d} = 0.39(3)$ form a scaling regime spanning a decade and a half. (b) The distribution function in time, leading to a growth exponent of $\beta_{2d} = 0.27(3)$.

light stereoscopy. In 1d, the fronts of the rice as the pile is grown shows excellent agreement with the 1d KPZ universality class with exponents $\alpha = 0.48(3)$ and $\beta = 0.33(3)$ from a scaling-regime spanning more than two decades. Thus having established the KPZ nature of the system under study, we analyze the full 2d surface of the pile, where find a roughness exponent of $\alpha_{2d} = 0.39(3)$ and $\beta_{2d} = 0.27(3)$. This is consistent with numerical simulations for ballistic deposition models [18, 19] and puts a strong experimental constraint on the available results on 2d KPZ simulations. Our results are in good agreement though with the results of Amar and Family [8], as well Bochaud and Cates [9] from numerical integration of the 2d KPZ equation. In addition however, we have studied the dependence of the exponent on the direction, where we find that the system is anisotropic with a somewhat higher exponent along the front direction. This is probably related to the difference between the two directions due to the growth mechanism.

1.6 Acknowledgements

This work was supported by the Swiss National Science Foundation and by FOM (Stiching voor Fundamenteel Onderzoek der Materie), which is financially supported by NWO (Nederlandse Organisatie voor Wetenschappelijk Onderzoek).

Bibliography

[1] Surdeanu, R., *et al.*, "Kinetic roughening of penetrating flux fronts in high T$_c$ superconducting thin fimls.", *Phys. Rev. Lett.* **83** (1999), 2054–2057.

[2] Maunuksela, J., *et al.*, "Kinetic roughening in slow combustion of paper", *Phys. Rev. Lett.* **79** (1997), 1515–1518; Myllys, A., *et al.*, "Kinetic roughening in slow combustion of paper", *Phys. Rev. E* **64** (2001), 036101.

[3] Welling, M.S., private communication.

[4] Ben-Jacob, E., *et al.*, "Communication, regulation and control during complex patterning of bacterial colonies", *Fractals* **2** (1994), 15–44.

[5] He, S., G.L.M.K.S. Kahanda, and P.-Z. Wong, "Roughness of wetting fluid invasion fronts in porous media", *Phys. Rev. Lett.* **69** (1992), 3731–3734.

[6] Barabasi, A.-L., and H.E. Stanley *Fractal Concepts in Surface Growth*, Cambridge University Press (1995).

[7] Kardar, M., G. Parisi, and Y.-C. Zhang, "Dynamic scaling of growing interfaces", *Phys. Rev. Lett.* **56** (1986), 889–892.

[8] Amar, J.G., and F. Family, "Numerical solution of a continuum equation for interface growth in 2+1 dimension", *Phys. Rev. A* **41** (1990), 3399–3402.

[9] Bouchaud, J.P., and M.E. Cates, "Self-consistent approach to the KPZ equation", *Phys. Rev. E* **47** (1993), R1455–R1458.

[10] Chakrabarti, A., and R. Toral, "Numerical study of a model for interface growth.", *Phys. Rev. B* **40** (1989), 11419–11421.

[11] Bak, P., C. Tang, and K. Wiesenfeld, "Self-organized criticality: An explanation of 1/f noise.", *Phys. Rev. Lett.* **59** (1987), 381–384 and "Self-organized criticality.", *Phys. Rev. A* **38** (1988), 364. See also Bak, P., *How nature works* (Oxford Univ. Press, 1995).

[12] Szabo, G.J., M.J. Alava, and J. Kertesz, "Self-organized criticality in the Kardar-Parisi-Zhang-equation", cond-mat/0112297; Alava, M.J., and K.B. Lauritsen, "Quenched noise and over-active sites in sandpile dynamics", *Europhys. Lett.* **53** (2001), 569–572.

[13] Frette, V., *et al.*, "Avalanche dynamics in a pile of rice", *Nature (London)* **379** (1996), 49–51.

[14] Günther, R., *Reconstruction and Roughening of two-dimensional granular surfaces*, Masters Thesis, Vrije Universiteit (2002).

[15] Zhang, Z., and G. Xu, *Epipolar Geometry in Stereo, Motion and Object Recognition: A unified Approach*, Kluwer Academic Publishers, (1996).

[16] Schmittbuhl, J., J.-P. Vilotte, and S. Roux, "Reliability of self-affine measurements", *Phys. Rev. E* **51** (1995), 131; Siegert, M., "Determining exponents in models of kinetic surface roughening", *ibid.* **53** (1996), 2309; Lopez, J.L., M.A. Rodriguez, and R. Cuerno, *ibid.* **56** (1997), 3993.

[17] KIM, J.M., and J.M. Kosterlitz, "Growth in a restricted solid-on-solid model.", *Phys. Rev. Lett.* **62** (1989), 2289–2292.

[18] BAIOD, R., *et al.*, "Dynamical scaling of the surface of finite-density ballistic aggregation.", *Phys. Rev. A* **38** (1988), 3672–3678.

[19] MEAKIN, P., *et al.*, "Ballistic deposition on surfaces.", *Phys. Rev. A* **34** (1986), 5091–5103.