Topics on the Equivalence Principle

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Abstract An update on the experimental bounds on the validity of the Einstein Equivalence Principle (EEP) and Strong Equivalence Principle (SEP) is presented. This work treats the three pillars of the EEP as well, i.e. the Weak Equivalence Principle (WEP), the Local Lorentz Invariance (LLI) and the Local Position Invariance (LPI). The most stringent limits on some quantities parametrizing a possible breakdown of the pillars are: $\eta = (-1 \pm 9_{\text{stat}} \pm 9_{\text{syst}}) \times 10^{-15}$[47] for the Eötvös ratio (WEP), $|\delta m_{\text{I}}^{ij}|/m_{\text{I}} < 3 \times 10^{-34}$[1] for the anomalous inertial mass tensor (LLI) and $\alpha = (-2.7 \pm 4.9) \times 10^{-7}$[33] for the gravitational-redshift anomaly (LPI), while for the SEP the gravitational-inertial mass ratio $1 + \Delta = 1 + (-3 \pm 5) \times 10^{-14}$[16] is taken into account. These values are just some of the empirical results regarding the formulations of the Equivalence Principle (EP). This report tries to provide a summary of the most recent experiments and of the future ones that aim to improve today's bounds, along with theoretical considerations with the objective of clarifying the relevance of a potential violation of the EP. These points are followed by a discussion on Schiff's conjecture, which raises arguments on the possibility that the WEP could imply the EEP.

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Introduction

The different formulations of the Equivalence Principle (EP) allow physicists to develop different theories. In most physics courses theories are built on their assumptions. In this paper instead the focus is put on the experimental efforts to bound their validity: a breakdown of one of them inevitably leads to the invalidation of some theories that had not been falsified before. As long as the Weak Equivalence Principle (WEP), also known as Universality of Free Fall, is regarded, the "Eötvös ratio" η is the signal of a possible breakdown. η gives information on the difference between gravitational and inertial mass. The best bound so far was obtained by the Microscope mission[47], at the level of 10^{-14}, but other proposed experiments may improve this result by 2-3 orders of magnitude[26][34][32][41]. When the Local Position Invariance (LPI) is considered, the most interesting parameter is α, which estimates deviations from the gravitational redshift predicted by the General Theory of Relativity, and it has been constrained to the level of 10^{-6}[33][13]. Other tests are planned[23][42] or currently run[21][31]. When the Local Lorentz Invariance (LLI) is taken into account however, this work cannot be regarded as complete as for the previous two topics, because in the framework of the Standard Model Extension[19] there are several parameters that have been bounded and treating all of them goes beyond the scope of this report. These three assumptions are also called the "pillars" of the Einstein Equivalence Principle (EEP): if one of them breaks down, the EEP and GR break down with it. If the EEP is extended to gravitational experiments, the Strong Equivalence Principle (SEP) is obtained. In this regard one of the most important quantities is the "Nordtvedt ratio" η_N, which parametrizes a possible difference between gravitational and inertial mass for self-gravitating bodies as a result of their gravitational energy. The smallest values for it are of order 10^{-4}[16][10] and further projects seek the 10^{-5} threshold[5][43]. Interesting data in the strong regime were shown by the analysis of the PSRJ0337+1715 pulsar triple system[38][2][50][45]. Finally, Schiff's conjecture is dealt with, i.e. whether the WEP implies the EEP. Depending on the case, or better on the theory, there are arguments supporting the conjecture for some of these theories and arguments contradicting it for others. Given the plethora of theories, a general proof or disproof of the conjecture has only a relative meaning, and will probably be never exhibited. An overview of these argumentations is offered in Section 4.1. As an additional point, in Appendix B a speculative connection between a SEP violation and the experimental tension on the Hubble constant H_0 is proposed, hinging on a possible time variation of Newton's constant G.

This work is mostly based on Clifford Will's book, *Theory and Experiment in Gravitational Physics*[54], on the article of Di Casola, Liberati and Sonego, *Nonequivalence of equivalence principles*[7] and on Moosbrugger's Master Thesis, *An update on the impact of Schiff's conjecture on tests of the Einstein Equivalence Principle today*[24].

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1 Formulations of the Equivalence Principle

There are many formulations of the Equivalence Principle and each of them allows different physical theories to be enunciated. In the following paragraphs the four most "famous" or used ones are reported.

1.1 The Weak Equivalence Principle (WEP)

There are two ways to enunciate the WEP. The first one is

$$m_{\mathrm{I}} = m_{\mathrm{P}}. \tag{1.1.1}$$

Here the equality of inertial and passive gravitational mass is assumed. Since every physical theory must have Newtonian Mechanics as its limit, by Newton's third law the active and passive gravitational masses are equal up to a universal factor, which can be set to 1 via a unit redefinition.
The second way is the following:

> *The trajectory of a test particle able to interact only gravitationally and with negligible self-gravity is independent of the particle internal structure and composition.*

Some points have to be clarified:

- *Interacts only gravitationally*: If other fields are present and the particle interacts with them, it is easy to imagine that a different trajectory will be the described by the test particle.
- *Test particle*: A particle that does not back-react with the surrounding environment. If not so, the particle changes the environment such that, theoretically, the same experiment cannot be replicated with another particle.
- *Negligible self-gravity*: Two quantities are suitable to describe self-gravitation:

$$\sigma_1 = \frac{2Gm}{c^2 r} = \frac{r_S}{r} \tag{1.1.2}$$

$$\sigma_2 = \frac{\Omega}{mc^2} \tag{1.1.3}$$

  with r_S the Schwarzschild radius, r the object radius and Ω its gravitational energy. The exact form of Ω will be defined later in Section 3 while discussing the possible violations of the SEP. As yet only the intuitive idea of σ_2 is needed: only objects unable to be held together just by gravity are taken into account. Regarding the first parameter, it portrays the space-time curvature around the object, and for black holes or neutron stars σ_1 is of order 1. Here the limits for $\sigma_1, \sigma_2 \to 0$ are considered.

The last two points are independent of one another: a micro black hole with a very small mass can be considered a test particle but is self-gravitating[7], while in a laboratory a rigid body of human scale is usually not a test particle and is not self-gravitating.

The WEP is also referred to as "Universality of Free Fall" (UFF).

1.2 The Einstein Equivalence Principle (EEP)

This was exploited by Einstein to devise his General Theory of Relativity. It can be stated as follows:

> *The outcome of any non-gravitational experiment of fundamental physics is locally unaffected by the presence of gravitational fields.*

Concepts to be clarified are present here as well:

- *Locally unaffected*: In a region of space-time small enough such that the gravitational field inhomogeneities can be neglected.
- *Fundamental physics:* For composite objects there are quantities that are always 0 in Special Relativity but are not 0 in the presence of a gravitational field, even locally[7]. Therefore the treatment is restricted.

As far as it is known, without gravity physics is Lorentz invariant. Consequently, the EEP can be regarded as consisting of:

- The WEP, already mentioned.
- The Local Lorentz Invariance (LLI): The results of non-gravitational experiments of fundamental physics are not affected by the velocity of the experimental apparatus.
- The Local Position Invariance (LPI): The results of non-gravitational experiments of fundamental physics are not affected by where and when they are performed.

It can be shown[54] that these "3 pillars" of the EEP lead to theories of gravity where space-time is endowed with a metric to which all non-gravitational fields couple in the same way (universal coupling), and where free falling trajectories are goedesics of this metric, such as in GR.

1.3 The Gravitational Weak Equivalence Principle (GWEP)

This is the analogue of the WEP for any σ_1 and σ_2, in other words obtained by relaxing the hypothesis of negligible self-gravitation:

> *In the void, the trajectory of a test particle able to interact only gravitationally is independent of the particle internal structure and composition.*

Adding *In the void* was necessary here, because a particle with a relevant self-gravitation disrupts the surrounding matter when not in the void, resulting in the infeasibility to replicate experiments.

1.4 The Strong Equivalence Principle (SEP)

This can been seen as the "extension" of the EEP to gravitational experiments, or:

> *The outcome of any experiment of fundamental physics is locally unaffected by the presence of gravitational fields.*

The three relative pillars here are the GWEP, the LLI (also for gravitational experiments) and the LPI (also for gravitational experiments).

Considering the resulting geometrical structure of space-time, the EEP allows the existence of additional gravitational fields that do not however couple with matter fields. There are some (non rigorous) arguments[54] suggesting that if a theory respects the SEP, the only gravitational field allowed is the metric ("metric theories"), and so far only GR was found to respect the SEP. Therefore it seems that

$$\text{SEP} \Rightarrow \text{GR}. \tag{1.4.1}$$

2 Tests of the EEP

It has been argued that the validity of the aforementioned principles are pivotal for the construction of different gravitational theories. The possibility of one of them to be violated opens thus doors for new physics. For this reason much experimental effort is put into determining bounds for their validity.

2.1 Tests of the WEP

Table 1: Summary of the most stringent bounds on the Eötvös ratio η

Experiment	Year	Reference	Method	Bound on η
Eöt-Wash (Be-Ti)	2012	[51]	rotating torsion balance	$(0.3 \pm 1.8) \times 10^{-13}$
Eöt-Wash (Be-Al)	2012	[51]	rotating torsion balance	$(-0.7 \pm 1.3) \times 10^{-13}$
Microscope [a]	2016	[47]	free fall in orbit	$(-1 \pm 9_{\text{stat}} \pm 9_{\text{syst}}) \times 10^{-15}$
Galileo Galilei [b]	-	[26]	rotating torsion balance in orbit	$< 10^{-17}$
STEP [b]	-	[34]	free fall in orbit	$< 10^{-18}$
SR-POEM [b]	-	[32]	free fall in a sounding rocket	$< 10^{-17}$

[a] The Microscope mission aims to $|\eta| < 10^{-15}$. Further data analyses are being carried out[41].
[b] These experiments are planned.

From a theoretical point of view, given the composition of an object, the interactions responsible for holding the object together could contribute in different ways to the inertial and passive gravitational mass of the system, such that

$$m_{\text{P}} = m_{\text{I}} + \sum_A \eta_A \frac{E_A}{c^2} \tag{2.1.1}$$

with A indexing the interaction, E_A the corresponding energy and η_A a number measuring the strength of the violation. For each interaction A, a formula for the corresponding E_A is not always know, and sometimes it is only semiempirical[54]. Listing these formulas or data goes beyond the scope of this work.[1] Using Newton's second law, one has

$$a_i = \left(\frac{m_{\text{P}}}{m_{\text{I}}}\right)_i g = \left[1 + \left(\sum_A \eta_A \frac{E_A}{m_{\text{I}} c^2}\right)_i\right] g \tag{2.1.2}$$

where i indexes a different particle. As Galileo Galilei observed, masses fall with the same acceleration in the same gravitational field. Considering two different particles, in order to find discrepancies to this statement the "Eötvös ratio" can be considered:

$$\eta \equiv \frac{2(a_1 - a_2)}{(a_1 + a_2)} \simeq \sum_A \eta_A \left[\left(\frac{E_A}{m_{\text{I}} c^2}\right)_1 - \left(\frac{E_A}{m_{\text{I}} c^2}\right)_2\right]. \tag{2.1.3}$$

A famous experiment to test the WEP was devised by Eötvös and his collaborators[9]. They used a torsion balance, i.e. two masses at the ends of a rod hanging thanks to a wire. The wire is not parallel to $\vec{g}$ because of the centrifugal acceleration caused by the rotation of the Earth. As a result a torque is applied on the rod, modulated in 24 hours. From this, information on η can be inferred. In modern versions with rotating torsion balances (Eöt-Wash experiments), e.g. by Wagner et al.[51], bounds on η of 1.4×10^{-13} and 1.8×10^{-13} for masses of (Berillium, Aluminum) and (Berillium, Titanium) were respectively obtained (at 1 σ confidence).

The Microscope mission yielded the best bound on the Eötvös parameter so far[47]: $|\eta| \leq 1.3 \times 10^{-14}$. The experimental setup consists of a free falling satellite, orbiting the Earth, with two test masses. By measuring the force required to keep the masses in relative equilibrium, data on η are acquired. The aim of the mission however is to bound $|\eta| < 10^{-15}$. Information about future mission-results will be found on the relative website[41]. There are other experiments that are programmed to be performed in space. The experiment Galileo Galilei[26]

[1] They can be however found in Moosbrugger[24].

is expected to test the WEP at the 10^{-17} scale by means of a rotating torsion balance in a satellite. For the experiment STEP[34] a satellite is also going to be employed, but it will contain two test masses whose relative acceleration will be measured. Another project whose objective is to reach better bounds on η than the ones already available is SR-POEM[32]. It will conduct a free fall test in a sounding rocket to achieve $|\eta| < 10^{-17}$.

For the interested reader, the historical development and further tests of the WEP are collected in the work of Nobili and Anselmi[27].

2.2 Test of the LLI

Table 2: Summary of the most stringent bounds on LLI violation parameters

Experiment type	Year	Reference	Parameter	Method	Bound
Hughes-Drever	2014	[1]	$\frac{\delta m_{\mathrm{I}}^{ij}}{m_{\mathrm{I}}}$	^{3}He-^{129}Xe spin precession	$< 3 \times 10^{-34}$
Kennedy-Thorndike	2010	[46]	$1 - \frac{c_0^2}{c^2}$ [a]	modulation w.r.t. CMB [b]	$(-4.8 \pm 3.7) \times 10^{-8}$
Light dispersion relation	2001	[11]	$f^{(3)}$	distant sources radiation	$< 2 \times 10^{-4}$ [c]

[a] Here a boost dependence variation is meant.
[b] Comparisons between hydrogen masers and sapphire oscillators were considered.
[c] No declared uncertainty can be found in the article, therefore it must be interpreted as an absolute upper bound.

An effect of a possible break down of the LLI is an anisotropy of the inertial mass, which could lead to different dynamics depending on the direction. This effect is formalized with the inertial mass tensor:

$$\delta m_{\mathrm{I}}^{ij} = \sum_A \delta_A^{ij} \frac{E_A}{c^2}, \tag{2.2.1}$$

where δ_A^{ij} gives the numerical and directional dependence of the violation as a function of the form of energy. A way to interpret the presence of δ_A^{ij} is to attribute it to preferred frame effects. The frame in which the Cosmic Microwave Background is homogeneous is often thought to be the preferred one. Given the Earth speed w with respect to this frame (369 km/s[35]), one expects

$$\delta_A^{ij} = \left(\frac{w}{c}\right)^l \delta_0^{ij}, \tag{2.2.2}$$

where δ_0^{ij} is called the "bare violation" of the LLI and l is a number. This quantity varies from theoretical model to theoretical model. In the so called "c^2 formalism" δ_0^{ij} is $1 - c_0^2/c^2$, where c is the limiting speed in the preferred frame and c_0 the limiting speed in an arbitrary one. Notwithstanding this, in the literature $\delta m_{\mathrm{I}}^{ij}/m_{\mathrm{I}}$ is often considered as the LLI violation.

Values of $\delta m_{\mathrm{I}}^{ij}/m_{\mathrm{I}}$ can be yielded by Hughes-Drever experiments[17][8]. In their original papers they considered a nucleus of ^{7}Li, whose ground state has angular momentum $I = 3/2$ split into a quadruplet by a magnetic field. In case any anisotropy is present, the transition frequencies between these levels differ slightly. The most recent experiment[1], that inferred data from the spin precession of coupled $^3\mathrm{He} - {}^{129}\mathrm{Xe}$ atoms in a homogeneous magnetic field, produced a bound on $\delta m_{\mathrm{I}}^{ij}/m_{\mathrm{I}}$ of 3×10^{-34}.
Other quantities may signal a LLI violation, for example the boost dependence of $1 - c_0^2/c^2$. One way to set empirical limits on it is via Kennedy-Thorndike experiments. The main idea of this type of experiments is similar to the Michelson-Morley experiment[18]: there is an interferometer but with an arm shorter than the other. This setup can bound the limiting-velocity dependence on the boost caused by the motion of the Earth. The most recent experiment[46] affirms this parameter is less than 6×10^{-8}.
Other possibilities for LLI violations comprehend a non linear dispersion relation for light. This can be

parametrized as

$$E_\gamma^2 = p_\gamma^2 c^2 + \sum_{n=1}^{\infty} f^{(n)} p_\gamma^n c^n E_{\rm Pl}^{2-n}, \tag{2.2.3}$$

$$\text{where} \quad E_{\rm Pl} = \sqrt{\frac{\hbar c^5}{G}} \sim 1.96 \times 10^9 \text{ J} \quad \text{is the Planck energy.} \tag{2.2.4}$$

In the work of Gleiser and Kozameh[11] the following dispersion relation was assumed:

$$E_\gamma^2 = p_\gamma^2 c^2 + f^{(3)} \frac{p_\gamma^3 c^3}{E_{\rm Pl}} = E^2_{\gamma_{\rm Lorentz}} \left(1 + f^{(3)} \frac{p_\gamma}{p_{\rm Pl}}\right). \tag{2.2.5}$$

Analyzing the spectrum of distant cosmological sources they obtained $f^{(3)} < 2 \times 10^{-4}$.

As it can be seen in Table 2, these experiments are quite dated, because modern tests of the LLI are carried out in the framework of the Standard Model Extension[54], with hundreds of parameters which are empirically constrained. More details on the SME, further tests of the LLI and recent updates are provided by the selected references: [19], [22] and [3].

2.3 Tests of the LPI

Table 3: Summary of the most stringent bounds on the LPI violation parameter α for the gravitational redshift

Experiment	Year	Reference	Method	Bound on α
GPA	1980	[48][49]	1 clock on the ground, 1 in a rocket	$(0.01 \pm 0.05 \pm 1.4) \times 10^{-4}$
Galileo	2018	[6]	clocks on satellites on elliptic orbits	$(0.19 \pm 2.48) \times 10^{-5}$
Galileo	2018	[15]	clocks on satellites on elliptic orbits	$(4.5 \pm 3.1) \times 10^{-5}$
Peil et. al	2013	[33]	null redshift[a] between ^{87}Rb and ^{1}H	$(-2.7 \pm 4.9) \times 10^{-7}$
Guéna et. al	2012	[13]	null redshift[a] between ^{133}Cs and ^{87}Rb	$(0.11 \pm 1.04) \times 10^{-6}$
ACES[b]	-	[23][42]	clocks in the ISS	$< 3 \times 10^{-6}$
RadioAstron[c]	now	[21][31]	like GPA, but on an elliptic orbit	$< 10^{-5}$

[a] Here a differential test on α is meant: the shown limit refers to $|\alpha_{\rm atom_1} - \alpha_{\rm atom_2}|$.
[b] This experiment is planned. The reported limit on α is the aim of the experiment.
[c] This test is currently being performed. The reported bound is the objective of the mission. So far only $\alpha = -0.016 \pm 0.003_{\rm stat} \pm 0.030_{\rm syst}$ was inferred.

A standard but at the same time important test of the LPI regards the gravitational redshift. The redshift is defined as

$$z \equiv \frac{\Delta\nu}{\nu}, \tag{2.3.1}$$

where ν is measured in two different positions. Assuming the validity of the WEP and the LLI but not of the LPI, considering two clocks in position A and B, in the approximation of weak gravitational fields, the following expression holds:[2]

$$c^2 {\rm d}\tau^2 = g^2(U(x^\mu_{\rm A})) \left[c^2 \left(1 + \frac{U(x^\mu_{\rm A})}{c^2}\right) {\rm d}t^2_{\rm A} - {\rm d}\vec{r}^2_{\rm A}\right] = g^2(U(x^\mu_{\rm B})) \left[c^2 \left(1 + \frac{U(x^\mu_{\rm B})}{c^2}\right) {\rm d}t^2_{\rm B} - {\rm d}\vec{r}^2_{\rm B}\right]. \tag{2.3.2}$$

Here U is the gravitational potential, x^μ_i the set of coordinates at position i, and given a photon ${\rm d}\tau$ can be interpreted as its time period measured in a reference frame where[3] $g(U) = 1$ and ${\rm d}t_i$ is its period measured in the

[2] U was used in Eq. (2.3.2) to break the LPI because U itself depends on position, and it is a quantity related to gravitational effects.

[3] For a spherical potential, this reference frame is usually at infinity. For a general one, the gauge can be changed redefining U: the position at which $U = 0$ can be chosen to have $g(U) = 1$.

frame i. Imagining that the photon is emitted at A and absorbed at B, at $\mathcal{O}(\Delta U/c^2)$ with $\Delta U = U(\mathrm{B}) - U(\mathrm{A})$, it is found that

$$z = (1+\alpha)\frac{\Delta U}{c^2}, \quad \text{with} \quad \alpha = \left.\frac{\mathrm{d}g}{\mathrm{d}U}\right|_{\mathrm{A}}. \tag{2.3.3}$$

Experiments measuring this effect were already performed at Harvard University[37] in 1965 with a limit on α of 10^{-2}. However one of the best current limits on α is provided by the Gravity Probe A experiment[48]: two hydrogen masers were employed, one in a spacecraft and one on the ground, and their time was compared after the trip of the former: α was found to be[49] $\sim 10^{-4}$. In 2014 two satellites of "Galileo", the European navigation system, were lunched into elliptic orbits instead of circular ones by accident. Two independent analysis[6][15] of the atomic clocks on board inferred α respectively a factor of 5 and 4 smaller compared to the GPA results.

The clocks in the foregoing experiments were taken along different trajectories or were placed in different positions. In what are called "null redshift tests" clocks of different composition are kept in the same place in order to test for a possible variation of α with the composition of the clock, while the Earth motion modulates U at the clocks position.[4] The two smallest limits on $|\alpha_1 - \alpha_2|$ obtained by this kind of tests[33][13] are of order 10^{-6}.

Other experiments are planned or being carried out in these years. For the ACES experiment [23] [42] atomic clocks will be placed in the International Space Station, with the aim of verifying the LPI down to 3×10^{-6}. An experiment that instead is already taking place is RadioAstron[21][31]: the idea is the same of GPA, but the clocks in the spacecraft are on an elliptic orbit. This should allow to bound $\alpha < 10^{-5}$, but so far only the order 10^{-2} was reached.

2.3.1 Is time invariance broken?

Hitherto the LPI was discussed as if it were a mere spatial topic; on the contrary it also deals with time. Recent efforts have focused on circumscribing how much quantities that are called "fundamental constants" vary with time. If variations were measured, the LPI would be violated. C. Will's book[54] offers a thorough report of many experiments on this topic. Variations of the fine structure constant, the weak interaction constant and the electron-proton mass ratio were inferred to be respectively less than 10^{-16}, 10^{-11} and 10^{-14} per year. Dealing with this lies outside the scope of this work, but the book does not mention the possible variation of the Hubble constant H_0, maybe because it is a topic that has emerged only in the last couple of years, or maybe because it is not a fundamental constant. For the curios reader a divulgative article was written by Adam Riess[39]. Linking the H_0 tension to a SEP breakdown is a topic that is going to be introduced in Appendix B. Given that it is inherently linked to gravitational experiments, it is recommended to read the next section on the violations of the SEP first.

3 Tests of the GWEP and the SEP

As it was done before, this sections aims to introduce and to list experiments defining validity bounds for the SEP. The theoretical discussion is really technical. The author of this work does not have the presumption to fully clarify it (and understand it) but tries however to shed some light on certain theoretical steps, so that the reader has an idea of what is being described. The theoretical arguments are mainly based on Will's book[54].

3.1 Theoretical overview

In 1967 Nordtvedt proposed the possibility that the gravitational energy might contribute in different ways to the inertial and gravitational mass[30] such that

$$\frac{m_{\mathrm{P}}}{m_{\mathrm{I}}} = 1 + \Delta \sim 1 + \eta_N \frac{\Omega}{m_{\mathrm{I}} c^2}, \quad \text{with } \Omega \text{ the gravitational energy.} \tag{3.1.1}$$

[4] If clock rates varied with position in a universal way, this phenomenon could be reabsorbed in the effects due to gravitation and would be thus immeasurable.

η_N is call the "Nordtvedt parameter" and the author also suggested methods to measure it[28], e.g. via lunar laser ranging, i.e. the earth-lunar distance is constantly monitored via laser beams reflected by mirrors on the Moon surface, looking for deviations from the expected trajectories. Subsequently it was proven[4] that in the Solar System higher order effects in $\Omega/m_{\rm I}c^2$ to Eq. (3.1.1) are irrelevant and that additional terms must be considered in calculations regarding strongly self-gravitating objects such as neutron stars.

Will's book shows that things are way more complicated than Eq. (3.1.1). To begin with, the framework is the PPN formalism with admitted violations of GR quantified by PPN parameters. Moreover, for bodies which have reached an internal equilibrium, i.e. whose internal structure does not depend on time, in general one has a tensorial equation of the form[5]

$$\left(\tilde{m}_{\rm I}\right)_a^{jk}\left(a_a^k\right)_{\rm Newt} = \left(\tilde{m}_{\rm P}\right)_a^{\ell m}\mathfrak{U}^{\ell m}_{,j} \quad \text{with} \quad \mathfrak{U}^{\ell m} \equiv \sum_{b\neq a}\left(\tilde{m}_{\rm A}\right)_b^{mq}\frac{n_{ab}^q n_{ab}^\ell}{r_{ab}}. \tag{3.1.2}$$

The quantities $\left(\tilde{m}_{\rm I}\right)_a^{jk}$, $\left(\tilde{m}_{\rm P}\right)_a^{jk}$ and $\left(\tilde{m}_{\rm A}\right)_a^{jk}$ are respectively the inertial, passive gravitational and active gravitational mass tensors of the object a, and the other terms in $\mathfrak{U}^{lm}$ give information on the relative distance of the objects taken into account. On top of that, $\left(a_a^j\right)_{\rm Newt}$, which is the equivalent of the Newtonian acceleration extended to this formalism, is not the only acceleration a body a experiences. In fact:

$$\vec{a}_a = \left(\vec{a}_a\right)_{\rm self} + \left(\vec{a}_a\right)_{\rm Newt} + \left(\vec{a}_a\right)_{\rm Nbody}. \tag{3.1.3}$$

$\left(\vec{a}_a\right)_{\rm self}$ is the self-acceleration and the PPN parameters from which it depends can be bounded by observing spinning pulsars. $\left(a_a\right)_{\rm Nbody}$ contains corrections arising from the objects not being pointlike and is responsible, among other things, for the "classical" perihelion shift of the planets.

The expressions for all of these quantities will be not provided but can be found of course in Will's book[54]. Nonetheless it is insightful that for spherically symmetric objects these accelerations as well as the mass tensors depend non trivially on the PPN-parameters, on the gravitational energy of the regarded bodies and on their total-energy mass. Proceeding in order, the PPN-parameters are named $\gamma, \beta, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3$, and ζ_4 with the constraint[53] $6\zeta_4 = 3\alpha_3 + 2\zeta_1 - 3\zeta_3$. Eq. (A.A.1) shows their role in the metric. In GR $\gamma = \beta = 1$ and the rest is 0. Updated bounds on these parameters are provided by the reference [53]. The gravitational energy of a body a is[6] [7]

$$\Omega_a \equiv -\frac{G}{2}\int_a \frac{\rho^*(\vec{x})\rho^*(\vec{x}')}{|\vec{x}-\vec{x}'|}{\rm d}^3x'{\rm d}^3x \quad \text{with} \quad \rho^* \equiv \rho\sqrt{-g}u^0. \tag{3.1.4}$$

ρ is the "standard" rest-mass-energy density but here ρ^* is considered because it satisfies the continuity equation in GR. Although the frame is not the one of GR, by considering ρ^* the equations turn out to be simpler.[8] In addition the inertial mass of a is

$$m_a \equiv \int_a \rho^*\left(1 + \frac{1}{2}|\vec{v} - \vec{v}_{a{\rm CM}}|^2 - \frac{1}{2}U^* + \Pi\right){\rm d}^3x \tag{3.1.5}$$

with $\vec{v}$ the velocity field of a sufficiently small volume of a, $\rho\Pi$ its density of internal energy (where Π is the internal energy per unit of mass) and

$$\vec{v}_{a{\rm CM}} \simeq \frac{1}{m_a}\int_a \rho^*\vec{v}{\rm d}^3x, \tag{3.1.6}$$

$$U^* \equiv G\int_a \frac{\rho^{*\prime}}{|\vec{x}-\vec{x}'|}{\rm d}^3x'. \tag{3.1.7}$$

$\left(\tilde{m}_{\rm I}\right)_a^{jk}$, $\left(\tilde{m}_{\rm P}\right)_a^{jk}$ and $\left(\tilde{m}_{\rm A}\right)_a^{jk}$ have different expressions and this constitutes a breakdown of the GWEP which is called "Nordtvedt effect". «Its existence does not violate the EEP or the Eötvös experiment, because the

[5] With $_{,j}$ the jth derivative is meant.
[6] u^0 is the 0th component of the proper velocity and g the determinant of the metric.
[7] In the following equations c is set to 1.
[8] As length and time, also simplicity is a relative concept.

laboratory-sized bodies considered in those situations have negligible self gravity, that is, $(\Omega/m)_{\text{ lab bodies }} < 10^{-39}$»(Will, [54], page 140). The explicit form for the mass tensors in the case of spherically symmetric objects reduce to three scalars and they are exhibited in Appendix A (see Eqs. (A.A.4) to (A.A.6)) but here it can be reported that at first order

$$\left(\frac{m_{\mathrm{P}}}{m_{\mathrm{I}}}\right)_a = 1 + \left(4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2\right)\frac{\Omega_a}{m_a} \tag{3.1.8}$$

and from Eq. (3.1.1) it follows that

$$\boxed{\eta_N = \left(4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2\right)\frac{\Omega_a}{m_a}.} \tag{3.1.9}$$

Considering a 3-body system, after toying with Eq. (3.1.2) one has 3 different terms: the newtonian acceleration, tidal effects and the Nordtvedt effect. Given two bodies, «[it] stretches or shrinks the orbit along a line directed toward the third [one]»(Will, [54], page 172). At $\mathcal{O}(\Omega/m)$ this effect is proportional to

$$\delta r \propto \eta_N \left(\frac{\Omega_1}{m_1} - \frac{\Omega_2}{m_2}\right). \tag{3.1.10}$$

Violations of the SEP cause preferred frames and preferred locations effects leading to orbit anomalies, the non constancy of G and anomalous spin precessions.

3.2 Experimental overview

Table 4: Summary of the most stringent bounds on the Nordtvedt parameter η_N, Δ (see Eq. (3.1.1)) and the time variation of Newton's constant

Experiment	Year	Reference	Method	Bounds on η_N	Bounds on Δ	Bounds on $\dot{G}/G$ [yr^{-1}]
Hoffman et al.[a]	2018	[16]	lunar ranging	$(-0.2 \pm 1.1) \times 10^{-4}$	$(-3 \pm 5) \times 10^{-14}$	$(7.1 \pm 7.6) \times 10^{-14}$
Messenger	2018	[10]	Mercury ephemeris	$(-6.6 \pm 7.2) \times 10^{-5}$	-	$< 4 \times 10^{-14}$
Gonzalez et al.	2011	[12]	pulsars timing	-	$< 4.6 \times 10^{-3}$ [b]	-
PSR J0337+1715	2018	[2]	pulsar timing	-	$< 2.6 \times 10^{-6}$	-
PSR J0337+1715	2020	[50]	pulsar timing	-	$(0.5 \pm 1.8) \times 10^{-6}$ [b]	-
BepiColombo[c]	-	[5][43]	Mercury ephemeris	$\approx 10^{-5}$	-	-

[a] The value for Δ is a differential one, i.e. $\Delta_{\mathrm{Earth}} - \Delta_{\mathrm{Moon}}$
[b] The uncertainty here is at 95% confidence level.
[c] This experiment is planned. The reported data are not results, rather the mission objectives.

The parameters Δ and η_N appearing in Eq. (3.1.1) and the quantity $\dot{G}/G$ have been inferred as yet in different ways. The most stringent bounds with lunar ranging experiments are[16] $\dot{G}/G = (7.1 \pm 7.6) \times 10^{-14}\mathrm{yr}^{-1}$, $\Delta_{\mathrm{EM}} = (-3 \pm 5) \times 10^{-14}$ and $\eta_N = (-0.2 \pm 1.1) \times 10^{-4}$. Here the subscript EM stands for $\Delta_{\mathrm{EM}} = \Delta_{\mathrm{Earth}} - \Delta_{\mathrm{Moon}}$. Jumping from the Earth-Moon system to the Solar one, the Messenger mission[10] measured accurately the position of Mercury over 7 years and obtained $\dot{G}/G < 4 \times 10^{-14}\mathrm{yr}^{-1}$ and $\eta_N = (-6.6 \pm 7.2) \times 10^{-5}$. Earlier data analyses[12] focused on millisecond pulsars inferred $\Delta < 4.6 \times 10^{-3}$ at 95% confidence level.
In 2014 the system of the pulsar PSR J0337+1715 was reported[38] to have been discovered. It is a triple system constituted by one pulsar and two white dwarfs with gravitational interactions in the strong field regime. Since these objects have ratios Ω/m ranging from 0.1 to 10^{-6} it was thought to be very informative on possible SEP breakdowns.[9] Two subsequent analyses in 2018[2] and 2020[50] provided values for Δ respectively of $< 2.6 \times 10^{-6}$

[9] Typical values of Ω/m in the Solar System for the Moon and the planets are $10^{-9} \div 10^{-11}$.

and $(0.5 \pm 1.8) \times 10^{-6}$ at 95% confidence level. Numerical studies[45] suggest that these limits can be decreased to 3×10^{-8}.
Further results are expected from the mission BepiColombo[5][43]; in particular $\eta_N \approx 10^{-5}$ was forecast as obtainable by the experimental apparatus.

4 Schiff's conjecture

So far the LLI, LPI and WEP have been treated separately, although Schiff had already argued[44] in 1960 that the three pillars of the EEP could/should be related. In his paper he discussed how classical GR tests actually support the theory and in a note at the very end of his article he wrote:«The Eötvös experiments show with considerable accuracy that the gravitational and inertial masses of matter are equal. This means that the ground state eigenvalue of the Hamiltonian for this matter appears equally in the inertial mass and in the interaction of this mass with a gravitational field. It would be quite remarkable if this could occur without the entire Hamiltonian being involved in the same way, in which case a clock composed of atoms whose motions are determined by this Hamiltonian would have its rate affected in the expected manner by a gravitational field.» In other words, as explained by Will[54] at page 35, «[..]the same mathematical formalism that produces equations describing the free fall of a hydrogen atom in a gravitational field must also produce equations that determine the energy levels of hydrogen in that gravitational field, and thereby determine the ticking rate of a hydrogen maser clock. Hence a violation of EEP in the fundamental machinery of a theory that manifests itself as a violation of WEP might also be expected to lead to a violation of Local Position Invariance». Here it is tried to be conveyed that it might be true that

$$\text{WEP} \Rightarrow \text{EEP}^{10}. \tag{4.0.1}$$

This is known as Schiff's conjecture. In his article, Haugan[14] collected and devised arguments supporting the conjecture.

4.1 Theoretical overview

In this work only two of Haugan's Gedankenexperimente are going to be reported. The first one works as follows. The binding energy of a composite body in a EEP-violating theory can be written at the lowest order in v^i and U^{ij} as

$$E_B(\vec{x},\vec{v}) = E_B^{(0)} + \delta m_{\mathrm{P}}^{ij} U^{ij}(\vec{x}) - \frac{1}{2}\delta m_{\mathrm{I}}^{ij} v^i v^j \tag{4.1.1}$$

$$\text{with} \quad U^{ij}(\vec{x}) = G\int \rho^*(\vec{x}')\frac{(x-x')^i(x-x')^j}{|\vec{x}-\vec{x}'|^3}\mathrm{d}^3x' \tag{4.1.2}$$

where ρ^* was defined in Eq. (3.1.4). Moreover the total energy of a composite body can be written, at the lowest order, as

$$E_{\mathrm{TOT}} = \left[M_0c^2 - E_B(\vec{x},\vec{v})\right] - M_R U(\vec{x}) + \frac{1}{2}M_R v^2 \tag{4.1.3}$$

with $U = \delta^{ij}U^{ij}$, M_0 the sum of the constituent masses and $M_R = M_0 - E_B(\vec{x},\vec{v})/c^2$ the rest mass of the body. Let this body be initially at $\vec{x} = \vec{h}$ with $v = 0$. Its energy is, neglecting second order terms,

$$E_{\mathrm{TOT}} = M_0c^2 - E_B^{(0)} - \left[M_0c^2 - E_B^{(0)}\right]\frac{U(\vec{h})}{c^2} - \delta m_{\mathrm{P}}^{ij}U^{ij}(\vec{h}). \tag{4.1.4}$$

The body now falls to $\vec{x} = 0$ with $\vec{v} = \vec{v}_f$. Its energy is

$$E_{\mathrm{TOT}} = M_0c^2 - E_B^{(0)} - \left[M_0c^2 - E_B^{(0)}\right]\frac{U(\vec{0})}{c^2} - \delta m_{\mathrm{P}}^{ij}U^{ij}(\vec{0}) + \frac{1}{2}\delta m_{\mathrm{I}}^{ij}v_f^i v_f^j + \frac{1}{2}\left[M_0 - \frac{E_B^{(0)}}{c^2}\right]v_f^2. \tag{4.1.5}$$

[10] This statement as well as the next argumentations can be analogously adapted for the GWEP and the SEP.

It is also assumed that U and E_B do not vary much during the fall, so that the acceleration of the body $\vec{a}$ is constant at lowest order. Therefore at lowest order

$$v_f^2 = -2\vec{a} \cdot \vec{h}. \tag{4.1.6}$$

Since energy must be conserved, the difference between Eq. (4.1.4) and Eq. (4.1.5) must be 0. In other words

$$\left[M_0 c^2 - E_B^{(0)}\right] \frac{U(\vec{h}) - U(\vec{0})}{c^2} + \delta m_{\mathrm{P}}^{ij} \left[U^{ij}(\vec{h}) - U^{ij}(\vec{0})\right] + \frac{1}{2}\delta m_{\mathrm{I}}^{ij} v_f^i v_f^j = -\frac{1}{2}\left[M_0 - \frac{E_B^{(0)}}{c^2}\right] v_f^2. \tag{4.1.7}$$

At first order $v_f^i v_f^j = -2a^i h^j$. In addition, given that the gravitational potential does not vary much, the difference in U can be substituted with the gradient at $\mathcal{O}\left(\vec{h}\right)$. On top of that, at first order $M_R c^2 \sim M_0 c^2 - E_B^{(0)}$. All in all

$$\vec{a} \cdot \vec{h} = \vec{\nabla} U(\vec{0}) \cdot \vec{h} + \frac{\delta m_{\mathrm{P}}^{ij}}{M_R} \vec{\nabla} U^{ij}(\vec{0}) \cdot \vec{h} - \frac{\delta m_{\mathrm{I}}^{ij}}{M_R} a^i h^j. \tag{4.1.8}$$

Rewriting these terms by realizing that $\vec{a} \sim \vec{g}$ at $\mathcal{O}\left(\vec{h}\right)$ and by defining $\vec{\nabla} U(\vec{0}) \equiv \vec{g}$ one has

$$\boxed{a^k = g^k + \frac{\delta m_{\mathrm{P}}^{ij}}{M_R} \frac{\partial U^{ij}(\vec{0})}{\partial x^k} - \frac{\delta m_{\mathrm{I}}^{ik}}{M_R} g^i.} \tag{4.1.9}$$

With this equation it is clear that

$$(\neg\mathrm{EEP} \Rightarrow \neg\mathrm{WEP}) \iff (\mathrm{WEP} \Rightarrow \mathrm{EEP}) \tag{4.1.10}$$

and the quantities

$$\frac{\delta m_{\mathrm{P}}^{ij}}{M_R} \quad \text{and} \quad \frac{\delta m_{\mathrm{I}}^{ik}}{M_R} \tag{4.1.11}$$

parametrize respectively the LPI and the LLI violation.

The second argument is based on energy conservation as well. Taking into account Eq. (4.1.1) and Eq. (4.1.3) one can consider at lowest order the energy of a body due to its compositeness. It is

$$E_C(\vec{x}, \vec{v}) = E_B(\vec{x}, \vec{v}) - E_B^{(0)} \frac{U(\vec{x})}{c^2} + \frac{1}{2} E_B^{(0)} \frac{v^2}{c^2}. \tag{4.1.12}$$

For $U = v = 0$ it reduces to the binding energy. In order to measure E_C one has to compare it with the ${}^S E_C(\vec{x}, \vec{v})$ of a standard system and to multiply it by some energy scale. This standard system may differ depending on the particular choice. Here another instance of the same system (with $v = 0$ w.r.t. the lab. frame and generally in another position) is taken to be the standard system, with the aim of having the same functional dependence for E_B. At this point, the natural energy scale that arises is $E_B^{(0)}$. Put into formulas

$$E_{\mathrm{measured}} = \frac{E_C(\vec{x}_1, \vec{v})}{E_C(\vec{x}_0, \vec{0})} E_B^{(0)} = \frac{E_B(\vec{x}_1, \vec{v}) - \frac{1}{c^2} E_B^{(0)} U(\vec{x}_1) + \frac{1}{2c^2} E_B^{(0)} v^2}{E_B(\vec{x}_0, \vec{0}) - \frac{1}{c^2} E_B^{(0)} U(\vec{x}_0)} E_B^{(0)}. \tag{4.1.13}$$

How could one measure this energy? E_C can be assumed to be energy of a photon necessary to break down the object into its constituents, which in case of an EEP-breakdown equals the binding energy only with $U = v = 0$. It can be written therefore that

$$E_\gamma = \frac{E_B^{(0)} + \delta m_{\mathrm{P}}^{ij} U^{ij}(\vec{x}_1) - \frac{1}{2}\delta m_{\mathrm{I}}^{ij} v^i v^j - \frac{1}{c^2} E_B^{(0)} U(\vec{x}_1) + \frac{1}{2c^2} E_B^{(0)} v^2}{E_B^{(0)} + \delta m_{\mathrm{P}}^{ij} U^{ij}(\vec{x}_0) - \frac{1}{c^2} E_B^{(0)} U(\vec{x}_0)} E_B^{(0)}. \tag{4.1.14}$$

Working out the foregoing expression to the leading order

$$\begin{aligned} E_\gamma &= E_B^{(0)} \left(1 + \frac{\delta m_{\mathrm{P}}^{ij} U^{ij}(\vec{x}_1)}{E_B^{(0)}} - \frac{\delta m_{\mathrm{I}}^{ij} v^i v^j}{2E_B^{(0)}} - \frac{U(\vec{x}_1)}{c^2} + \frac{v^2}{2c^2}\right)\left(1 - \frac{\delta m_{\mathrm{P}}^{ij} U^{ij}(\vec{x}_0)}{E_B^{(0)}} + \frac{U(\vec{x}_0)}{c^2}\right) \\ &= E_B^{(0)} \left(1 + \frac{\delta m_{\mathrm{P}}^{ij}}{E_B^{(0)}} \Delta U^{ij} - \frac{\delta m_{\mathrm{I}}^{ij}}{2E_B^{(0)}} v^i v^j - \frac{\Delta U}{c^2} + \frac{v^2}{2c^2}\right) \end{aligned} \tag{4.1.15}$$

where $\Delta U = U(\vec{x}_1) - U(\vec{x}_0)$ and likewise ΔU^{ij}. From this equation one has the redshift formula for the photon emitted by the system at $(\vec{x}_1, \vec{v})$ and measured by the system at $(\vec{x}_2, \vec{0})$[11]:

$$\boxed{z = \frac{E_{\text{measured}} - E_{\text{standard}}}{E_{\text{measured}}} = \frac{E_\gamma - E_B^{(0)}}{E_\gamma} = -\frac{\Delta U}{c^2} + \frac{\delta m_{\text{P}}^{ij}}{E_B^{(0)}} \Delta U^{ij} + \frac{v^2}{2c^2} - \frac{\delta m_{\text{I}}^{ij}}{2E_B^{(0)}} v^i v^j.} \tag{4.1.16}$$

Here the same result of Eq. (4.1.10) was found but the quantities parametrizing the LPI and the LLI violations (or in other words gravitational redshift and transverse redshift violations) are respectively

$$\frac{\delta m_{\text{P}}^{ij}}{E_B^{(0)}} \quad \text{and} \quad \frac{\delta m_{\text{I}}^{ij}}{2E_B^{(0)}}. \tag{4.1.17}$$

It is not as simple as this however: There are known theories of gravity in which Schiff's conjecture is false and it has only been proven in particular cases. As it can be read in the first page of their article, Lightman and Lee[20] proved Schiff's conjecture for «test bodies, made of electromagnetically interacting point particles, that fall from rest in a static, spherically symmetric gravitational field», in the framework of what is known as the "TH$\epsilon\mu$ formalism". Ni instead worked under weaker assumptions[25] and showed that for an EM interaction lagrangian density of the kind

$$\mathcal{L} = -\frac{1}{16\pi} \chi^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} - \sqrt{-g} j^\epsilon A_\epsilon - \sum_i m_i \frac{\mathrm{d}s_i}{\mathrm{d}t} \delta(x - x_i(t)) \tag{4.1.18}$$

Schiff's conjecture is valid if and only if

$$\chi^{\alpha\beta\gamma\delta} = \frac{\sqrt{-g}}{2} (g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\gamma} + \varphi \varepsilon^{\alpha\beta\gamma\delta}) \tag{4.1.19}$$

where φ is a scalar field and ε is the Levi-Civita tensor.

In this author's opinion the question "Is Schiff's conjecture valid?" has no value per sé, because any physicist could come up with a new theory for which it should be understood whether the conjecture is true or false. Focus should rather be put on the experimental verification of the EEP and, in case it came to light to be broken, on testing the pillars according to Eq. (4.1.9) and Eq. (4.1.16) to understand whether these relations are satisfied.

4.2 Experimental considerations

With the help of two Gedankenexperimente, already used by Nordtvedt[29] and Haugan[14] and put together by Moosbrugger[24], in the hypothesis of isotropic mass tensors δm_{I} and δm_{P} some useful relations among the breakdowns of the three EEP pillars can be calculated. The aim of this paragraph is to underline the hypotheses under which these relations (that can be found in [24]) are yielded in terms of their possible empirical falsification. Working with isotropic mass tensors is correct if one considers spherically symmetric objects. In addition to this, one of the two Gedankenexperimente considered by Moosbrugger (the one proposed by Nordtvedt) implies the possibility for bodies to absorb photons of arbitrary frequency and to interact to build up more composite objects. This certainly works theoretically and it also explains what the conjecture is about, but experimentally these hypotheses can be thought of as too restrictive[12]. The results shown in Eq. (4.1.9) and Eq. (4.1.16) are obtained under the only assumption of being able to accurately measure the binding energy of an object[13], which of course can be experimentally challenging, but is free of any constraint on the matter examined, and of course if one wants to estimate possible discrepancies one "only" has to perform Eötvös and redshift experiments. On the other hand, a drawback of this formalism would be that the relations among η, α and δ_A, defined respectively in Eq. (2.1.3), Eq. (2.3.3) and Eq. (2.2.1), would be more difficult to deduce than the ones in the

[11] Actually here the inverse process is assumed to happen: The photon arrives from $\vec{x}_1$ after the formation of the body travelling with velocity $\vec{v}$. It is not a problem: The energy required to decompose an object is the same freed by its composition process.

[12] The curious reader is invited to take a look at the works of Moosbrugger[24] and Nordtvedt[29].

[13] This assumption is also present in the work of Moosbrugger.

work of Moosbrugger. In fact in Eq. (4.1.9) and Eq. (4.1.16) the pillars violations are ratios between a mass tensor and either a binding energy (Eq. (4.1.9)) or a rest mass (Eq. (4.1.16)). Without any knowledge of further studies on the topic, these quantities would have to be linked case by case, material by material and shape by shape in order to have numerical bounds.

Conclusions

To the best of the physical community knowledge the SEP seems to hold only in GR, which implies the EEP. So far neither the SEP nor the pillars of the EEP have shown any sign of invalidity. Ranging experiments in the Solar System and in the Earth-Moon system yielded no evidence for a SEP breakdown, as well as the accurate timing of the PSRJ0337+1715 pulsar. Further analyses of the Microscope data[41], as well as future experiments such as Galileo Galilei[26], STEP[34], SR-POEM[32], ACES[23][42], RadioAstron[21][31], BepiColombo[5][43] and accelerator physics experiments may shed more light on the EP, but as yet there is no indication that GR is falsified because of an EEP breakdown. Gendankenexperimente about Schiff's conjecture were also discussed, even though some of them were not explicitly reported in this work. The ones of Haugan[14] seem to be more general than the one of Nordtvedt[29], but it is the latter one that can lead to an easier mathematical connection between violations of the EEP pillars, albeit under restrictive hypotheses. These relations can be found in the work of Moosbrugger[24]. An interesting question is whether these relations can be generalized to include the results arising from the Gedankenexperimente of Haugan, which are included in this work and deal with weaker assumptions. If one considers the theoretical framework in which the conjecture is formulated, one realizes that in some frameworks the conjecture holds while in others it does not. One could argue that in the end the important task is to mathematically link EEP-pillar-breakdowns via the conjecture, test the EEP and, in case of violations, test Schiff's conjecture.

Appendix

A Additional quantities for SEP tests

In the PPN formalism

$$\begin{aligned} g_{00} &= -1 + 2U + 2\left(\psi - \beta U^2\right) + O\left(\epsilon^3\right) \\ g_{0j} &= -\frac{1}{2}\left[4(1+\gamma) + \alpha_1\right] V_j - \frac{1}{2}\left[1 + \alpha_2 - \zeta_1 + 2\xi\right] X_{,0j} + O\left(\epsilon^{5/2}\right) \\ g_{jk} &= (1 + 2\gamma U)\delta_{jk} + O\left(\epsilon^2\right) \end{aligned} \tag{A.A.1}$$

with

$$\begin{aligned} \psi :=& \frac{1}{2}\left(2\gamma + 1 + \alpha_3 + \zeta_1 - 2\xi\right)\Phi_1 - \left(2\beta - 1 - \zeta_2 - \xi\right)\Phi_2 + \left(1 + \zeta_3\right)\Phi_3 \\ &+ \left(3\gamma + 3\zeta_4 - 2\xi\right)\Phi_4 - \frac{1}{2}\left(\zeta_1 - 2\xi\right)\Phi_6 - \xi\Phi_W \end{aligned} \tag{A.A.2}$$

and

$$U(t, \vec{x}) \equiv G \int \frac{\rho^*\left(t, \vec{x}'\right)}{\left|\vec{x} - \vec{x}'\right|} \mathrm{d}^3x', \quad \nabla^2 U = -4\pi G \rho^* \tag{A.A.3}$$

The many Φ fields, X and V are functions of the pressure, internal energy and velocity fields of the object, as of course of its mass density ρ^* (see Eq. (3.1.4)). More details are shown in Will's book[54].

The inertial, passive gravitational and active gravitational mass for a spherical object at thermodynamical equilibrium is

$$m_{\mathrm{I}a} \equiv m_a\left[1 + \left(\alpha_1 - \frac{2}{3}\alpha_2 + \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2\right)\frac{\Omega_a}{m_a}\right], \tag{A.A.4}$$

$$m_{\mathrm{P}a} \equiv m_a\left[1 + \left(4\beta - \gamma - 3 - \frac{10}{3}\xi\right)\frac{\Omega_a}{m_a}\right], \tag{A.A.5}$$

$$m_{\mathrm{A}a} \equiv m_a\left[1 + \left(4\beta - \gamma - 3 - \frac{10}{3}\xi - \frac{1}{2}\alpha_3 - \frac{1}{3}\zeta_1 - 2\zeta_2\right)\frac{\Omega_a}{m_a} + \zeta_3\frac{E_a}{m_a} + \left(3\zeta_4 - \zeta_1 - \frac{3}{2}\alpha_3\right)\frac{P_a}{m_a}\right], \tag{A.A.6}$$

with

$$P_a \equiv \int_a p\mathrm{d}^3x \qquad \text{where } p \text{ is the pressure field and} \qquad E_a \equiv \int_a \rho^*\Pi\mathrm{d}^3x \qquad \text{(see Eq. (3.1.5))}. \tag{A.A.7}$$

B The Hubble tension and the SEP[14] [15]

The more distant a galaxy is from Earth the faster it moves away from it: This is estimated by the Hubble constant H_0. The data from the Planck collaboration[36] measured by observing the Cosmic Microwave Background suggest a value of $67.4 \pm 0.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Similar results were inferred by measuring the early Universe[39]. Measurements of the late Universe regarding for example Cepheid variables and type Ia supernovae[40] provide instead $74.03 \pm 1.42\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Other observations of the late Universe are compatible with this value[39].

It appears that predictions for the Universe expansion, which is a gravitational phenomenon, based on the early Universe contradict the ones looking at today's Universe. There are many hypothesis that are being studied with the aim to solve this tension, such as adding additional fields to the Einstein equations. However, here a speculative relation with the breakdown of the SEP is proposed: since the laws of physics in this case appear to be non-time-invariant, one might attribute the Hubble tension to a violation of the LPI for gravitational experiments. One might also advance the hypothesis that this violation is due to a non vanishing time derivative of Newton's constant, i.e. $\dot{G} \neq 0$. The Hubble parameter $H(t)$ is a function of time describing the variation of

[14] This subsection is far from being a complete treatment of the subject and it is only a summary. Please read the article of Adam Riess[39].

[15] The Hubble tension was already mentioned in Section 2.3.1.

the Hubble constant; H_0 is just $H(t)$ evaluated at the present time. In light of the Friedman equations, $H^2(t)$ has three terms: one is due to the Universe curvature, another one arises from the cosmological constant and the last one is

$$\frac{8\pi G}{3c^2}\varepsilon(t), \qquad \text{with } \varepsilon \text{ the Universe energy density.} \tag{A.B.1}$$

It is clear that a variation of G leads to a different Universe evolution. Is there a way to link the Hubble tension to $\dot{G}/G$? Wand and Chen[52] worked with the observations of the Planck collaboration (among data from other sources) and fitted these to a model with $G = G(t)$ as a free parameter. They divided the Universe lifetime in 3 epochs and allowed for different values of G in these epochs. They extrapolated a variation compatible with 0 and the tension was not solved. Nonetheless, to this author's knowledge, this is the first attempt to link the Hubble tension to a LPI breakdown of the SEP.

An interesting question is whether and how the tension could be fully formalized in terms of a falsifiable SEP breakdown.

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